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Supersymmetric solutions for non-relativistic holography

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Abstract: We construct families of supersymmetric solutions of type IIB and $D = 11$ supergravity that are invariant under the non-relativistic conformal algebra for various values of dynamical exponent $z \geq 4$ and $z \geq 3$, respectively. The solutions are based on five- and seven-dimensional Sasaki-Einstein manifolds and generalise the known solutions with dynamical exponent $z = 4$ for the type IIB case and $z = 3$ for the $D = 11$ case, respectively.

Keywords: AdS-CFT Correspondence, M-Theory, Gauge-gravity correspondence

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Contents

1 Introduction

There has recently been much interest in finding holographic realisations of systems invariant under the non-relativistic conformal algebra starting with the work [1, 2] and discussed further in related work [3]–[32]. Such systems are invariant under Galilean transformations, generated by time and spatial translations, spatial rotations, Galilean boosts and a mass operator, which is a central element of the algebra, combined with scale transformations. If x^+ is the time coordinate, and $\mathbf{x}$ denotes d spatial coordinates, the scaling symmetry acts as

$$\mathbf{x} \to \mu \mathbf{x}, \qquad x^+ \to \mu^z x^+ , \tag{1.1}$$

where z is called the dynamical exponent. When $z = 2$ this non-relativistic conformal symmetry can be enlarged to an invariance under the Schrödinger algebra which includes an additional special conformal generator.

The solutions found in [1, 2] with $d = 2$ and $z = 2$ were subsequently embedded into type IIB string theory in [8–10] and were based on an arbitrary five-dimensional Sasaki-Einstein manifold, SE_5. The work of [9] also constructed type IIB solutions with $d = 2$ and $z = 4$ and again these were constructed using an arbitrary SE_5. It was also shown in [9] that the solutions with $z = 2$ and $z = 4$ can be obtained from a five dimensional theory with a massive vector field after a Kaluza-Klein reduction on the SE_5 space [9]. This procedure was generalised to solutions of $D = 11$ supergravity in [31]: using a similar KK reduction on an arbitrary seven-dimensional Sasaki-Einstein space, SE_7, solutions with non relativistic conformal symmetry with $d = 1$ and $z = 3$ were found.

The type IIB solution of [8–10] with $z = 2$ do not preserve any supersymmetry [9]. One aim of this note is to show that, by contrast, the type IIB solutions of [9] with $z = 4$ and the $D = 11$ solutions of [31] with $z = 3$ are both supersymmetric and generically preserve two supersymmetries. A second aim is to generalise both of these supersymmetric solutions to different values of z. We will construct new supersymmetric solutions using eigenmodes of the Laplacian acting on one-forms on the SE_5 or SE_7 space. If the eiegenvalue is μ then we obtain type IIB solutions with $z = 1+\sqrt{1+\mu}$ and $D = 11$ solutions with $z = 1+\frac{1}{2}\sqrt{4+\mu}$. This gives rise to type IIB solutions with $z \geq 4$ and $D = 11$ solutions with $z \geq 3$, respectively. For the case of S^5 we get solutions with $z = 4, 5, \ldots$ while for the case of S^7 we get solutions with $z = 3, 3\frac{1}{2}, 4, \ldots$ and both of these preserve 8 supersymmetries.

Our constructions have some similarities with the construction of type IIB solutions in [24] that were based on eigenmodes of the Laplacian acting on scalar functions on the SE_5 space. Our IIB solutions preserve the same supersymmetry and we show how our solutions can be superposed with those of [24] while maintaining a scaling symmetry. An analogous superposition is possible for the $D = 11$ solutions, which we shall also describe.

2 The type IIB solutions

The ansatz for the type IIB solutions we shall consider is given by

$$\begin{aligned} ds^2 &= \frac{dr^2}{r^2} + r^2\left[2dx^+dx^- + dx_1^2 + dx_2^2\right] + ds^2(SE_5) + 2r^2 C dx^+ \\ F_5 &= 4r^3 dx^+ \wedge dx^- \wedge dr \wedge dx_1 \wedge dx_2 + 4Vol(SE_5) \\ &\quad - dx^+ \wedge \left[*_{CY_3} dC + d(r^4 C) \wedge dx_1 \wedge dx_2\right] \end{aligned} \tag{2.1}$$

where SE_5 is an arbitrary five-dimensional Sasaki-Einstein space and the metric $ds^2(SE_5)$ is normalised so that the Ricci tensor is equal to four times the metric (i.e. the same normalisation as that of a unit radius five-sphere). Recall that the metric cone over the SE_5,

$$ds^2(CY_3) = dr^2 + r^2 ds^2(SE_5)\,, \tag{2.2}$$

is Calabi-Yau. The Kähler form on the CY_3 is denoted ω_{ij} and the complex structure is defined[1] by $J_i{}^j = \omega_{ik}g^{kj}$, where g_{ij} is the Calabi-Yau cone metric. We will define the one-form η, which is dual to the Reeb vector on SE_5 by

$$\eta_i = -J_i{}^j\,(d\log r)_j \ . \tag{2.3}$$

The one-form C is a one-form on the CY_3 cone. When $C = 0$ we have the standard $AdS_5 \times SE_5$ solution of type IIB which, in general, preserves eight supersymmetries (four Poincaré and four superconformal), corresponding to an $N = 1$ SCFT in $d = 4$. More generally, we can deform this solution by choosing $C \neq 0$ provided that dC is co-closed on CY_3:

$$d *_{CY} dC = 0\ . \tag{2.4}$$

With this condition, F_5 is closed and in fact it is also sufficient for the type IIB Einstein equations to be satisfied. As we will show these solutions preserve one half of the Poincaré supersymmetries. Note that the solution is invariant under the transformation

$$x^- \to x^- - \Lambda, \qquad C \to C + d\Lambda \tag{2.5}$$

[1] While this is standard in the physics literature, often in the maths literature $J_i{}^j = -\omega_{ik}g^{kj}$.

for some function Λ on the CY cone. Thus, if $dC = 0$, we can remove C, at least locally, by such a transformation.

We will look for solutions where the one-form C has weight λ under the action of $r\partial_r$. Then it is straightforward to check, following [1] and [2] that our solution is invariant under non-relativistic conformal transformations with two spatial dimensions x^1, x^2 and dynamical exponent $z = 2 + \lambda$. For example the scaling symmetry is acting as in (1.1) combined with $r \to \mu^{-1} r$, $x^- \to \mu^{2-z} x^-$. Following the analysis of closed and co-closed two forms on cones (such as dC) in appendix A of [33] we consider solutions constructed from a co-closed one-form β on the SE_5 space that is an eigenmode of the Laplacian $\Delta_{SE} = (d^\dagger d + dd^\dagger)_{SE}$:

$$C = r^\lambda \beta, \qquad \Delta_{SE}\beta = \mu\beta, \qquad d^\dagger \beta = 0 \ . \tag{2.6}$$

It is straightforward to check that dC is co-closed providing that $\mu = \lambda(\lambda + 2)$. For our applications we choose the branch $\lambda = -1 + \sqrt{1 + \mu}$ leading to solutions with

$$z = 1 + \sqrt{1 + \mu} \ . \tag{2.7}$$

A general result valid for any five-dimensional Einstein space, normalised as we have, is that for co-closed 1-forms $\mu \geq 8$ and $\mu = 8$ holds iff the 1-form is dual to a Killing vector (see section 4.3 of [34]). Thus in general our construction leads to solutions with

$$z \geq 4 \ . \tag{2.8}$$

Since all SE_5 manifolds have at least the Reeb Killing vector, dual to the one-form η, this bound is always saturated. Indeed the solution of [9] with $z = 4$ is in our class. Specifically it can be obtained by setting $C = \sigma r^2 \eta$ (and redefining $x^- \to -x^-/2$): one can explicitly check that η is co-closed on SE_5 and is an eigenmode of Δ_{SE} with eigenvalue $\mu = 8$. Note that for this solution the two-form dC is proportional to the Kähler-form of the Calabi-Yau cone: $dC = 2\sigma\omega$.

On S^5 the spectrum of Δ_{S^5} acting on one-forms is well known and we have $\mu = (s+1)(s+3)$ for $s = 1, 2, 3\ldots$ (see for example [35] eq (2.20)) leading to $\lambda = s + 1$ and hence new classes of solutions with $z = 4, 5, 6\ldots$. Note that these solutions come in families, transforming in the SO(6) irreps **15**, **64**, **175**, ... To obtain similar results for $T^{1,1}$ one can consult [36].

We now discuss a construction that can be used when the spectrum of the Laplacian acting on functions is known, but not acting on one-forms. For example, the scalar Laplacian was studied in [40] for the $Y^{p,q}$ metrics [41], but as far as we know it has not been discussed acting on one-forms. Specifically we construct $(1, 1)$ forms dC on the CY cone using scalar functions Φ on the cone as follows. We write

$$C_i = J_i{}^j \partial_j \Phi \tag{2.9}$$

for some function Φ on CY_3. A short calculation shows that if

$$\nabla^2_{CY}\Phi = \alpha \tag{2.10}$$

for some constant α then dC is co-closed. The two-form dC is a $(1,1)$ form on CY_3 and it is primitive, $J^{ij}dC_{ij} = 0$, if and only if $\alpha = 0$. Observe that the solution of [9] with $z = 4$ fits into this class by taking $\Phi = -\sigma r^2/2$ and $\alpha = -6\sigma$, leading to $C = \sigma r^2 \eta$.

We now consider solutions with $\alpha = 0$, corresponding to harmonic functions[2] on the CY cone with dC $(1,1)$ and primitive. We next write

$$\Phi = r^\lambda f \tag{2.11}$$

where f is a function on the SE_5 space satisfying

$$-\nabla^2_{SE_5} f = kf \tag{2.12}$$

with $k = \lambda(\lambda+4)$ (see e.g. [37]). For the solutions of interest we choose the branch $\lambda = -2 + \sqrt{4+k}$ leading to $z = \sqrt{4+k}$. For the special case of the five-sphere we can check with the results that we obtained above. The eigenfunctions f on the five-sphere are given by spherical harmonics with $k = l(l+4)$, $l = 1, 2, \ldots$ and hence $z = l + 2$. The $l = 1$ harmonic appears to violate the bound (2.8). However, it is straightforward to see that the construction for $l = 1$ leads to $dC = 0$ for which C can be removed by a transformation of the form (2.5). Thus for S^5 we should consider $l \geq 2$ leading to solutions with $z = 4, 5, \ldots,$ as above. It is worth pointing out that for higher values of l some of the eigenfunctions will also lead to closed C: if we consider the harmonic function on $\mathbb{R}^6$ given by $x^{i_1} \ldots x^{i_l} c_{i_1 \ldots i_l}$ where c is symmetric and traceless then, with $J = dx^1 \wedge dx^2 + dx^3 \wedge dx^4 + dx^5 \wedge dx^6$ we see that $dC = 0$ if $J_{[i}{}^{j} c_{k]j i_3 \ldots i_l} = 0$.

2.1 Supersymmetry

We introduce the frame

$$\begin{aligned}
e^+ &= r dx^+ \\
e^- &= r(dx^- + C) \\
e^2 &= r dx_1 \\
e^3 &= r dx_2 \\
e^4 &= \frac{dr}{r} \\
e^m &= e^m_{SE}, \qquad m = 5, \ldots, 9
\end{aligned} \tag{2.13}$$

where e^m_{SE} is an orthonormal frame for the SE_5 space. We can write

$$F_5 = B_5 + *_{10} B_5 \tag{2.14}$$

$$B_5 = 4e^+ \wedge e^- \wedge e^2 \wedge e^3 \wedge e^4 - re^+ \wedge dC \wedge e^2 \wedge e^3 \tag{2.15}$$

where we have chosen $\epsilon_{+-23456789} = +1$. The Killing spinor equation can be written

$$D_M \epsilon + \frac{i}{16} \not{F} \Gamma_M \epsilon = D_M \epsilon + \frac{i}{2} \not{B} \Gamma_M \epsilon = 0\ . \tag{2.16}$$

[2]Note that in general the one-form C defined in (2.9) has a component in the dr direction, unlike in (2.6). However, locally we can remove it by a transformation of the form (2.5). Also, one can directly show that the resulting one-form β is co-closed on the SE_5 space.

We are using the conventions for type IIB supergravity [42, 43] as in [44] and in particular, $\Gamma_{11} = \Gamma_{+-23456789}$ with the chiral IIB spinors satisfying $\Gamma_{11}\epsilon = -\epsilon$.

If ϵ are the Killing spinors for the $AdS_5 \times SE_5$ solution, then we find that we must also impose that

$$\begin{aligned} \Gamma^{+-23}\epsilon &= i\epsilon \\ \Gamma^{+}\epsilon &= 0\ . \end{aligned} \tag{2.17}$$

The first condition maintains the Poincaré supersymmetries but breaks all of the superconformal supersymmetries (this can be explicitly checked using, for example, the results of [45]). The second condition breaks a further half of these[3]. Thus when $dC \neq 0$, we preserve two Poincaré supersymmetries for a generic SE_5 and this is increased to eight Poincaré supersymmetries for S^5.

3 The $D = 11$ solutions

The construction of the $D = 11$ solutions is very similar. We consider the ansatz for D=11 supergravity solutions:

$$\begin{aligned} ds^2 &= \frac{d\rho^2}{4\rho^2} + \rho^2\left[2dx^+dx^- + dx^2\right] + ds^2(SE_7) + 2\rho^2 C dx^+ \\ G &= -3\rho^2 dx^+\wedge dx^-\wedge d\rho\wedge dx + dx^+\wedge dx\wedge d(\rho^3 C) \end{aligned} \tag{3.1}$$

where SE_7 is a seven-dimensional Sasaki-Einstein space and $ds^2(SE_7)$ is normalised so that the Ricci tensor is equal to six times the metric (this is the normalisation of a unit radius seven-sphere). It is convenient to change coordinates via $\rho = r^2$ to bring the solution to the form

$$\begin{aligned} ds^2 &= \frac{dr^2}{r^2} + r^4\left[2dx^+dx^- + dx^2\right] + ds^2(SE_7) + 2r^4 C dx^+ \\ G &= -6r^5 dx^+\wedge dx^-\wedge dr\wedge dx + dx^+\wedge dx\wedge d(r^6 C)\ . \end{aligned} \tag{3.2}$$

In these coordinates the cone metric

$$ds^2_{CY} = dr^2 + r^2 ds^2(SE_7) \tag{3.3}$$

is a metric on Calabi-Yau four-fold. We will use the same notation for the CY space as in the previous section.

When the one-form C is zero we have the standard $AdS_4 \times SE_7$ solution of $D = 11$ supergravity that, in general, preserves eight supersymmetries. We again find that all the equations of motion are solved if C is a one-form on CY_4 and the two-form dC is co-closed

$$d *_{CY} dC = 0\ . \tag{3.4}$$

[3]That we preserve the Poincaré supersymmetries suggests that we can extend our solutions away from the near horizon limit of the D3-branes. This is indeed the case but we won't expand upon that here.

The solutions are again invariant under the transformation (2.5). We will consider solutions where the one-form C has weight λ under the action of $r\partial_r$, corresponding to dynamical exponent $z = 2 + \lambda/2$. As before, using the results in appendix A of [33], we consider solutions constructed from a co-closed one-form β on the SE_7 space that is an eigenmode of the Laplacian Δ_{SE}:

$$C = r^\lambda \beta, \qquad \Delta_{SE}\beta = \mu\beta, \qquad d^\dagger\beta = 0\ . \tag{3.5}$$

One can check that dC is co-closed providing that $\mu = \lambda(\lambda + 4)$. For our applications we choose the branch $\lambda = -2 + \sqrt{4 + \mu}$ leading to solutions with

$$z = 1 + \frac{1}{2}\sqrt{4+\mu}\ . \tag{3.6}$$

A general result valid for any seven-dimensional Einstein space, normalised as we have, is that for co-closed 1-forms $\mu \geq 12$ and $\mu = 12$ holds iff the 1-form is dual to a Killing vector (see section 4.3 of [34]). Thus in general our construction leads to solutions with

$$z \geq 3 \tag{3.7}$$

and the bound is again saturated for all SE_7 spaces. Observe that the solutions of [31] with $z = 3$ fit into this class. Specifically they are obtained by setting $C = \sigma r^2\eta$ (after redefining $x \to x/2$ and $x^- \to -x^-/8$). On S^7 the spectrum of Δ_{S^7} is well known and we have $\mu = s(s+6)+5$ for $s = 1, 2, 3\ldots$ (see for example [34] eq (7.2.5)) leading to $\lambda = 1+s$ and hence new classes of solutions with $z = 3, 3\frac{1}{2}, 4, \ldots$. These solutions come in families transforming in the $SO8)$ irreps $\mathbf{28}$, $\mathbf{160_v}$, $\mathbf{567_v}$, ... Results on the spectrum of the Laplacian on some homogeneous SE_7 spaces can be found in [46–48].

As before we can construct $(1,1)$ co-closed two-forms dC using scalar functions Φ on CY_4 We write

$$C_i = J_i{}^j\partial_j\Phi, \qquad \nabla^2_{CY}\Phi = \alpha\ \ . \tag{3.8}$$

and dC is again primitive if and only if $\alpha = 0$. The solutions of [31] with $z = 3$ arise by taking $\Phi = \sigma r^2$ and $\alpha = -8\sigma$ leading to $C = \sigma r^2\eta$. We now focus on solutions with $\alpha = 0$, corresponding to harmonic functions on the CY cone. We take

$$\Phi = r^\lambda f \tag{3.9}$$

where f is a function on the SE_7 space satisfying

$$-\nabla^2_{SE_7} f = kf \tag{3.10}$$

with $k = \lambda(\lambda+6)$. For our applications we choose the branch $\lambda = -3 + \sqrt{9+k}$ leading to solutions with $z = \frac{1}{2} + \frac{1}{2}\sqrt{9+k}$. For example, on the seven-sphere the eigenfunctions f are given by spherical harmonics with $k = l(l+6)$ with $l = 1, 2, \ldots$ and hence $z = 2 + l/2$. Excluding the $l = 1$ harmonic, as it can be removed by a transformation of the form (2.5), for S^7 we are left with solutions with $z = 3, 7/2, 4, \ldots$, as above.

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3.1 Supersymmetry

We introduce a frame

$$\begin{aligned} e^+ &= r^2 dx^+ \\ e^- &= r^2(dx^- + C) \\ e^2 &= r^2 dx \\ e^3 &= \frac{dr}{r} \\ e^m &= e^m_{SE}, \qquad m = 4, \dots, 10 \ . \end{aligned} \tag{3.11}$$

We thus have

$$\begin{aligned} G &= 6e^+ \wedge e^- \wedge e^2 \wedge e^3 + r^2 e^+ \wedge e^2 \wedge dC \\ *_{11} G &= -6Vol(SE_7) + dx^+ *_{CY} dC \end{aligned} \tag{3.12}$$

where we have chosen the orientation $\epsilon_{+-23....10} = +1$.

The Killing spinor equation can be written as

$$\nabla_M \epsilon + \frac{1}{288}[\Gamma_M{}^{N_1N_2N_3N_4} - 8\delta^{N_1}_M \Gamma^{N_2N_3N_4}]G_{N_1N_2N_3N_4}\epsilon = 0 \ . \tag{3.13}$$

We are using the conventions for $D = 11$ supergravity [49] as in [50] and in particular $\Gamma_{+-2345678910} = +1$.

If ϵ are the Killing spinors arising for the $AdS_4 \times SE_7$ solution, then we find that we must also impose that

$$\begin{aligned} \Gamma^{+-2}\epsilon &= -\epsilon \\ \Gamma^+\epsilon &= 0 \ . \end{aligned} \tag{3.14}$$

The first condition maintains the Poincaré supersymmetries but breaks all of the superconformal supersymmetries. The second condition breaks a further half of these. Thus when $dC \neq 0$, we preserve two Poincaré supersymmetries for a generic SE_7 and this is increased to eight Poincaré supersymmetries for S^7.

3.2 Skew-Whiffed solutions

If $AdS_4 \times SE_7$ is a supersymmetric solution of $D = 11$ supergravity, then if we "skew-whiff" by reversing the sign of the flux (or equivalently changing the orientation of SE_7) then apart from the special case when the SE_7 space is the round S^7, all supersymmetry is broken [51]. Despite the lack of supersymmetry, such solutions are known to be perturbatively stable [51]. Similarly, if we reverse the sign of the flux in our new solutions (3.2), we will obtain solutions of $D = 11$ supergravity that will generically not preserve any supersymmetry.

4 Further generalisation

We now discuss a further generalisation of the solutions that we have considered so far, preserving the same amount of supersymmetry, which incorporate the construction of [24]. For type IIB the metric is now given by

$$ds^2 = \frac{dr^2}{r^2} + r^2\left[2dx^+dx^- + dx_1^2 + dx_2^2\right] + ds^2(SE_5) + r^2\left[2Cdx^+ + h(dx^+)^2\right] \quad (4.1)$$

with the five-form unchanged from (2.1). The conditions on the one-form C are as before and we demand that h is a harmonic function on the CY_3 cone:

$$\nabla^2_{CY} h = 0\ . \quad (4.2)$$

Choosing h to have weight λ' under $r\partial_r$ we take

$$h = r^{\lambda'} f'\,, \quad (4.3)$$

where f' is an eigenfunction of the Laplacian on SE_5 with eigenvalue k'

$$-\nabla^2_{SE_5} f' = k' f' \quad (4.4)$$

with $k' = \lambda'(\lambda' + 4)$. If we set $C = 0$ and choose the branch $\lambda' = -2 + \sqrt{4 + k'}$ then these are the solutions constructed in section 5 of [24] and have dynamical exponent $z = \frac{1}{2}\sqrt{4 + k'}$. As noted in [24] an application of Lichnerowicz's theorem [52, 53] implies that these solutions have $z \geq 3/2$ with $z = 3/2$ only possible for S^5. Now if there is a scalar eigenfunction with eigenvalue k' and a one-form eigenmode of the Laplacian on SE_5 with eigenvalue μ that satisfy $z = \frac{1}{2}\sqrt{4 + k'} = 1 + \sqrt{1 + \mu}$ then we can superpose the solution with h as in (4.3) and the one-form C as in (2.6) and have a solution with scaling symmetry with this value of z. For example on S^5, using the notation as before, we have $k' = l'(l'+4)$, $l' = 1, 2, \ldots$ and $\mu = (s+1)(s+3)$, $s = 1, 2, \ldots$ and hence we must demand that $l' = 2(s+2)$, $s = 1, 2, \ldots$, giving solutions with $z = 3 + s$.

The story for $D = 11$ is very similar. The metric is now given by

$$ds^2 = \frac{dr^2}{r^2} + r^4\left[2dx^+dx^- + dx^2\right] + ds^2(SE_7) + r^4\left[2Cdx^+ + h(dx^+)^2\right] \quad (4.5)$$

with the four-form unchanged from (3.2). The conditions on the one-form C are as before and we demand that h is a harmonic function on the CY_4 cone:

$$\nabla^2_{CY} h = 0\ . \quad (4.6)$$

Choosing h to have weight λ' under $r\partial_r$ we take

$$h = r^{\lambda'} f'\,, \quad (4.7)$$

where f' is an eigenfunction of the Laplacian on SE_7 with eigenvalue k'

$$-\nabla^2_{SE_7} f' = k' f' \quad (4.8)$$

with $k' = \lambda'(\lambda'+6)$. If we set $C = 0$ and chose the branch $\lambda' = -3 + \sqrt{9+k'}$ then these solutions have dynamical exponent $z = \frac{1}{4}(1+\sqrt{9+k'})$. Lichnerowicz's theorem [52, 53] implies that these solutions have $z \geq 5/4$ with $z = 5/4$ only possible for S^7. If there is a scalar eigenfunction with eigenvalue k' and a one-form eignemode of the Laplacian on SE_7 with eigenvalue μ that satisfy $z = \frac{1}{4}(1+\sqrt{9+k'}) = 1 + \frac{1}{2}\sqrt{4+\mu}$ then we can superpose the solution with h as in (4.7) and the one-form C as in (3.5) and have a solution with scaling symmetry with this value of z. For example on S^7, using the notation as before, we have $k' = l'(l'+6)$, $l' = 1, 2, \ldots$ and $\mu = s(s+6)+5$, $s = 1, 2, \ldots$ and hence we must demand that $l' = 2(s+3)$, $s = 1, 2, \ldots$, giving solutions with $z = \frac{1}{2}(5+s)$.

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